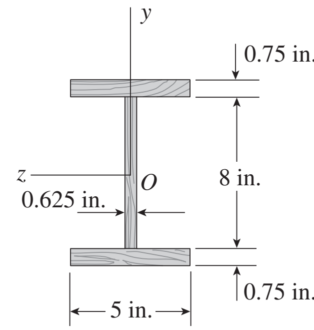
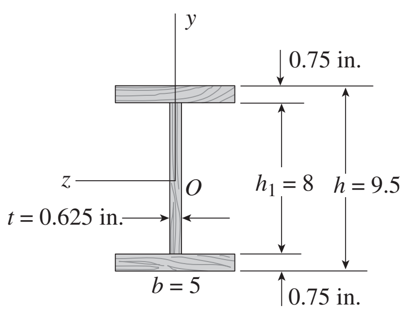
Built-Up Beams

Problem 5.11-1 A prefabricated wood I-beam serving as a floor joist has the cross section shown in the figure. The allowable load in shear for the glued joints between the web and the flanges is 65 lb/in. in the longitudinal direction.

Determine the maximum allowable shear force V_{max} for the beam.



Solution 5.11-1 **Wood I-beam**



All dimensions in inches.

Find V_{max} based upon shear in the glued joints.

Allowable load in shear for the glued joints is 65 lb/in.

$\therefore \quad f_{allow} = 65$ lb/in.

$$f = \frac{VQ}{I} \qquad V_{max} = \frac{f_{allow} I}{Q}$$

$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = \frac{1}{12}(5)(9.5)^3 - \frac{1}{12}(4.375)(8)^3$$

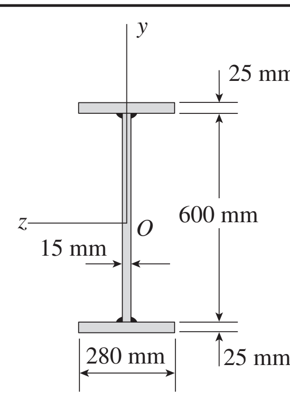
$$= 170.57 \text{ in.}^4$$

$$Q = Q_{flange} = A_f d_f = (5)(0.75)(4.375) = 16.406 \text{ in.}^3$$

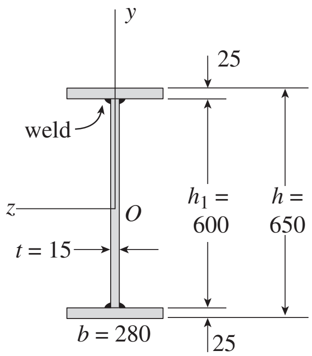
$$V_{max} = \frac{f_{allow} I}{Q} = \frac{(65 \text{ lb/in.})(170.57 \text{ in.}^4)}{16.406 \text{ in.}^3} = 676 \text{ lb} \quad \longleftarrow$$

Problem 5.11-2 A welded steel girder having the cross section shown in the figure is fabricated of two 280 mm × 25 mm flange plates and a 600 mm × 15 mm web plate. The plates are joined by four fillet welds that run continuously for the length of the girder. Each weld has an allowable load in shear of 900 kN/m.

Calculate the maximum allowable shear force V_{max} for the girder.



Solution 5.11-2 **Welded steel girder**



All dimensions in millimeters.

Allowable load in shear for one weld is 900 kN/m.

$\therefore\ f_{allow} = 2(900) = 1800$ kN/m

$$f = \frac{VQ}{I} \qquad V_{max} = \frac{f_{allow}I}{Q}$$

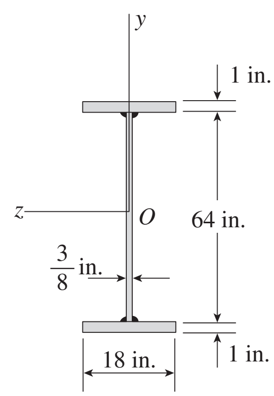
$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = \frac{1}{12}(280)(650)^3 - \frac{1}{12}(265)(600)^3$$
$$= 1638 \times 10^6 \text{ mm}^4$$

$$Q = Q_{flange} = A_f d_f = (280)(25)(312.5) = 2.1875 \times 10^6 \text{ mm}^3$$

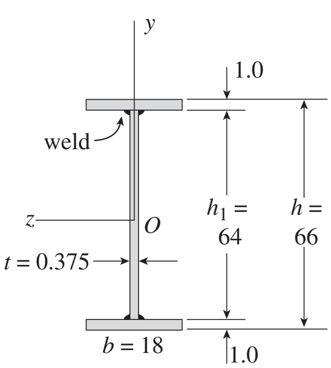
$$V_{max} = \frac{f_{allow}I}{Q} = \frac{(1800 \text{ kN/m})(1638 \times 10^6 \text{ mm}^4)}{2.1875 \times 10^6 \text{ mm}^3}$$
$$= 1.35 \text{ MN} \quad \longleftarrow$$

Problem 5.11-3 A welded steel girder having the cross section shown in the figure is fabricated of two 18 in. × 1 in. flange plates and a 64 in. × 3/8 in. web plate. The plates are joined by four longitudinal fillet welds that run continuously throughout the length of the girder.

If the girder is subjected to a shear force of 300 kips, what force *F* (per inch of length of weld) must be resisted by each weld?



Solution 5.11-3 **Welded steel girder**



All dimensions in inches.

$V = 300$ k
F = force per inch of length of one weld

$$f = \text{shear flow} \qquad f = 2F = \frac{VQ}{I} \qquad F = \frac{VQ}{2I}$$

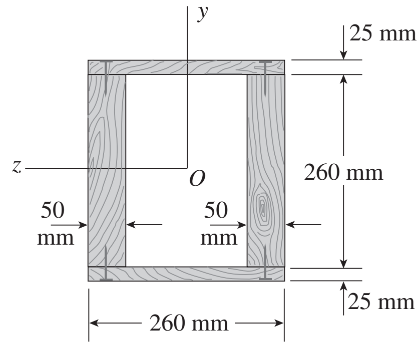
$$I = \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12} = \frac{1}{12}(18)(66)^3 - \frac{1}{12}(17.625)(64)^3$$
$$= 46{,}220 \text{ in.}^4$$

$$Q = Q_{flange} = A_f d_f = (18)(1.0)(32.5) = 585 \text{ in.}^3$$

$$F = \frac{VQ}{2I} = \frac{(300 \text{ k})(585 \text{ in.}^3)}{2(46{,}220 \text{ in.}^4)} = 1900 \text{ lb/in.} \quad \longleftarrow$$

Problem 5.11-4 A box beam of wood is constructed of two 260 mm × 50 mm boards and two 260 mm × 25 mm boards (see figure). The boards are nailed at a longitudinal spacing $s = 100$ mm.

If each nail has an allowable shear force $F = 1200$ N, what is the maximum allowable shear force V_{max}?



Solution 5.11-4 **Wood box beam**

All dimensions in millimeters.

$b = 260 \quad b_1 = 260 - 2(50) = 160$
$h = 310 \quad h_1 = 260$

s = nail spacing = 100 mm
F = allowable shear force for one nail = 1200 N
f = shear flow between one flange and both webs

$$f_{allow} = \frac{2F}{s} = \frac{2(1200\text{ N})}{100\text{ mm}} = 24\text{ kN/m}$$

$$f = \frac{VQ}{I} \qquad V_{max} = \frac{f_{allow}I}{Q}$$

$$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 411.125 \times 10^6\text{ mm}^4$$

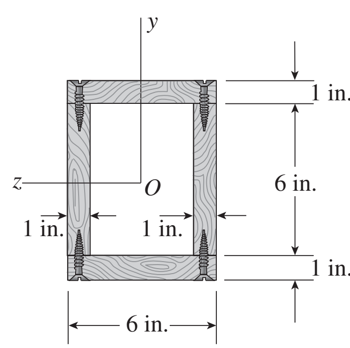
$$Q = Q_{flange} = A_f d_f = (260)(25)(142.5) = 926.25 \times 10^3\text{ mm}^3$$

$$V_{max} = \frac{f_{allow}I}{Q} = \frac{(24\text{ kN/m})(411.125 \times 10^6\text{ mm}^4)}{926.25 \times 10^3\text{ mm}^3}$$

$$= 10.7\text{ kN} \quad \longleftarrow$$

Problem 5.11-5 A box beam constructed of four wood boards of size 6 in. × 1 in. (actual dimensions) is shown in the figure. The boards are joined by screws for which the allowable load in shear is $F = 250$ lb per screw.

Calculate the maximum permissible longitudinal spacing s_{max} of the screws if the shear force V is 1200 lb.



Solution 5.11-5 **Wood box beam**

All dimensions in inches.

$b = 6.0 \quad b_1 = 6.0 - 2(1.0) = 4.0$
$h = 8.0 \quad h_1 = 6.0$
F = allowable shear force for one screw = 250 lb
V = shear force = 1200 lb
s = longitudinal spacing of the screws
f = shear flow between one flange and both webs

$$f = \frac{VQ}{I} = \frac{2F}{s} \qquad \therefore\ s_{max} = \frac{2FI}{VQ}$$

$$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 184\text{ in.}^4$$

$$Q = Q_{flange} = A_f d_f = (6.0)(1.0)(3.5) = 21\text{ in.}^3$$

$$s_{max} = \frac{2FI}{VQ} = \frac{2(250\text{ lb})(184\text{ in.}^4)}{(1200\text{ lb})(21\text{ in.}^3)}$$

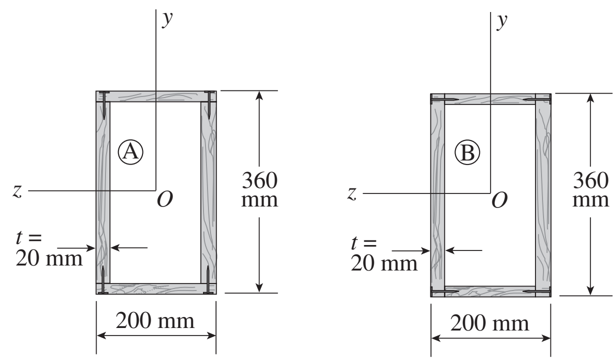
$$= 3.65\text{ in.} \quad \longleftarrow$$

Problem 5.11-6 Two wood box beams (beams A and B) have the same outside dimensions (200 mm $\times$ 360 mm) and the same thickness (t = 20 mm) throughout, as shown in the figure on the next page. Both beams are formed by nailing, with each nail having an allowable shear load of 250 N. The beams are designed for a shear force V = 3.2 kN.

(a) What is the maximum longitudinal spacing s_A for the nails in beam A?

(b) What is the maximum longitudinal spacing s_B for the nails in beam B?

(c) Which beam is more efficient in resisting the shear force?



Solution 5.11-6 **Two wood box beams**

Cross-sectional dimensions are the same.

All dimensions in millimeters.

$b = 200 \qquad b_1 = 200 - 2(20) = 160$
$h = 360 \qquad h_1 = 360 - 2(20) = 320$
$t = 20$
F = allowable load per nail = 250 N
V = shear force = 3.2 kN

$$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 340.69 \times 10^6 \text{ mm}^4$$

s = longitudinal spacing of the nails
f = shear flow between one flange and both webs

$$f = \frac{2F}{s} = \frac{VQ}{I} \qquad \therefore\ s_{\max} = \frac{2FI}{VQ}$$

(a) BEAM A

$$Q = A_p d_p = (bt)\left(\frac{h-t}{2}\right) = (200)(20)\left(\frac{1}{2}\right)(340)$$
$$= 680 \times 10^3 \text{ mm}^3$$

$$s_A = \frac{2FI}{VQ} = \frac{(2)(250 \text{ N})(340.7 \times 10^6 \text{ mm}^4)}{(3.2 \text{ kN})(680 \times 10^3 \text{ mm}^3)}$$
$$= 78.3 \text{ mm} \quad \longleftarrow$$

(b) BEAM B

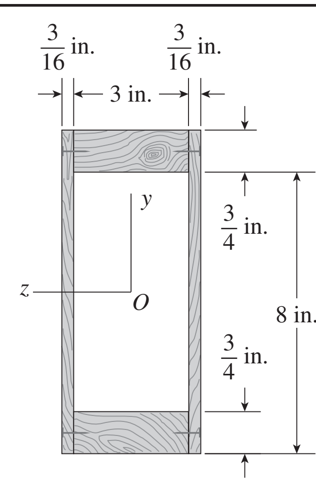
$$Q = A_f d_f = (b - 2t)(t)\left(\frac{h-t}{2}\right) = (160)(20)\frac{1}{2}(340)$$
$$= 544 \times 10^3 \text{ mm}^3$$

$$s_B = \frac{2FI}{VQ} = \frac{(2)(250 \text{ N})(340.7 \times 10^6 \text{ mm}^4)}{(3.2 \text{ kN})(544 \times 10^3 \text{ mm}^3)}$$
$$= 97.9 \text{ mm} \quad \longleftarrow$$

(c) BEAM B IS MORE EFFICIENT because the shear flow on the contact surfaces is smaller and therefore fewer nails are needed. ⟵

Problem 5.11-7 A hollow wood beam with plywood webs has the cross-sectional dimensions shown in the figure. The plywood is attached to the flanges by means of small nails. Each nail has an allowable load in shear of 30 lb.

Find the maximum allowable spacing s of the nails at cross sections where the shear force V is equal to (a) 200 lb and (b) 300 lb.



Solution 5.11-7 **Wood beam with plywood webs**

All dimensions in inches.

$b = 3.375 \quad b_1 = 3.0$
$h = 8.0 \quad h_1 = 6.5$
F = allowable shear force for one nail = 30 lb
s = longitudinal spacing of the nails
f = shear flow between one flange and both webs

$$f = \frac{VQ}{I} = \frac{2F}{s} \qquad \therefore\ s_{\max} = \frac{2FI}{VQ}$$

$$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 75.3438 \text{ in.}^4$$

$$Q = Q_{\text{flange}} = A_f d_f = (3.0)(0.75)(3.625) = 8.1563 \text{ in.}^3$$

(a) V = 200 lb

$$s_{\max} = \frac{2FI}{VQ} = \frac{2(30 \text{ lb})(75.344 \text{ in.}^4)}{(200 \text{ lb})(8.1563 \text{ in.}^3)}$$
$$= 2.77 \text{ in.} \leftarrow$$

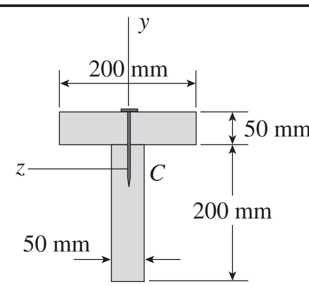
(b) V = 300 lb

By proportion,

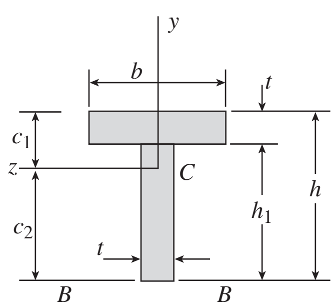
$$s_{\max} = (2.77 \text{ in.})\left(\frac{200}{300}\right) = 1.85 \text{ in.} \leftarrow$$

Problem 5.11-8 A beam of T cross section is formed by nailing together two boards having the dimensions shown in the figure.

If the total shear force V acting on the cross section is 1600 N and each nail may carry 750 N in shear, what is the maximum allowable nail spacing s?



Solution 5.11-8 **T-beam (nailed)**



All dimensions in millimeters.
V = 1600 N
F = allowable load per nail
F = 750 N
b = 200 mm $\quad t$ = 50 mm
h = 250 mm $\quad h_1$ = 200 mm
s = nail spacing
Find $s_{\max}$

LOCATION OF NEUTRAL AXIS (z AXIS)

Use the lower edge of the cross section (line B-B) as a reference axis.

$$Q_{BB} = (h_1 t)\left(\frac{h_1}{2}\right) + (bt)\left(h - \frac{t}{2}\right)$$
$$= (200)(50)(100) + (200)(50)(225)$$
$$= 3.25 \times 10^6 \text{ mm}^3$$

$$A = bt + h_1 t = t(b + h_1) = (50)(400)$$
$$= 20 \times 10^3 \text{ mm}^2$$

$$c_2 = \frac{Q_{BB}}{A} = \frac{3.25 \times 10^6 \text{ mm}^3}{20 \times 10^3 \text{ mm}^2} = 162.5 \text{ mm}$$

$$c_1 = h - c_2 = 250 - 162.5 = 87.5 \text{ mm}$$

MOMENT OF INERTIA ABOUT THE NEUTRAL AXIS

$$I = \frac{1}{3}tc_2^3 + \frac{1}{3}t(h_1 - c_2)^3 + \frac{1}{12}bt^3 + bt\left(c_1 - \frac{t}{2}\right)^2$$
$$= \frac{1}{3}(50)(162.5)^3 + \frac{1}{3}(50)(37.5)^3 + \frac{1}{12}(200)(50)^3$$
$$+ (200)(50)(62.5)^2$$
$$= 113.541 \times 10^6 \text{ mm}^4$$

FIRST MOMENT OF AREA OF FLANGE

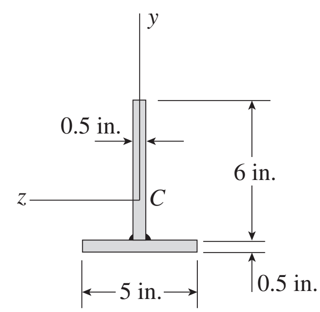
$$Q = bt\left(c_1 - \frac{t}{2}\right) = (200)(50)(62.5) = 625 \times 10^3 \text{ mm}^3$$

MAXIMUM ALLOWABLE SPACING OF NAILS

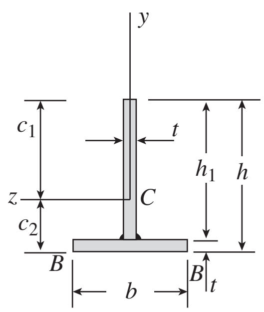
$$f = \frac{VQ}{I} = \frac{F}{s}$$

$$s_{\max} = \frac{F_{\text{allow}} I}{VQ} = \frac{(750 \text{ N})(113.541 \times 10^6 \text{ mm}^4)}{(1600 \text{ N})(625 \times 10^3 \text{ mm}^3)}$$
$$= 85.2 \text{ mm} \leftarrow$$

Problem 5.11-9 The T-beam shown in the figure is fabricated by welding together two steel plates. If the allowable load for each weld is 2.0 k/in. in the longitudinal direction, what is the maximum allowable shear force V?



Solution 5.11-9 **T-beam (welded)**



All dimensions in inches.

F = allowable load per inch of weld
$F = 2.0$ k/in.
$b = 5.0 \qquad t = 0.5$
$h = 6.5 \qquad h_1 = 6.0$
V = shear force
Find V_{max}

LOCATION OF NEUTRAL AXIS (z AXIS)

Use the lower edge of the cross section (line B-B) as a reference axis.

$$Q_{BB} = (bt)\left(\frac{t}{2}\right) + (h_1 t)\left(h - \frac{h_1}{2}\right)$$
$$= (5)(0.5)(0.25) + (6)(0.5)(3.5) = 11.25 \text{ in.}^3$$

$$A = bt + h_1 t = (5)(0.5) + (6)(0.5) = 5.5 \text{ in.}^2$$

$$c_2 = \frac{Q_{BB}}{A} = \frac{11.125 \text{ in.}^3}{5.5 \text{ in.}^2} = 2.0227 \text{ in.}$$
$$c_1 = h - c_2 = 4.4773 \text{ in.}$$

MOMENT OF INERTIA ABOUT THE NEUTRAL AXIS

$$I = \frac{1}{3}tc_1^3 + \frac{1}{3}t(c_2 - t)^3 + \frac{1}{12}bt^3 + (bt)\left(c_2 - \frac{t}{2}\right)^2$$
$$= \frac{1}{3}(0.5)(4.4773)^3 + \frac{1}{3}(0.5)(1.5227)^3 + \frac{1}{12}(5)(0.5)^3$$
$$+ (5)(0.5)(1.7727)^2 = 23.455 \text{ in.}^4$$

FIRST MOMENT OF AREA OF FLANGE

$$Q = bt\left(c_2 - \frac{t}{2}\right) = (5)(0.5)(1.7727) = 4.4318 \text{ in.}^3$$

SHEAR FLOW AT WELDS

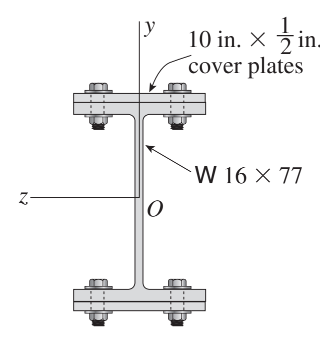
$$f = 2F = \frac{VQ}{I}$$

MAXIMUM ALLOWABLE SHEAR FORCE

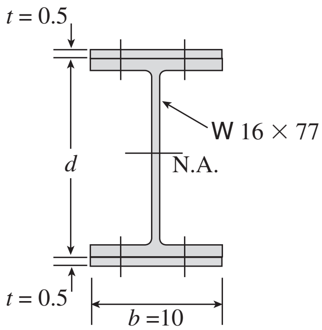
$$V_{max} = \frac{2FI}{Q} = \frac{2(2.0 \text{ k/in.})(23.455 \text{ in.}^4)}{4.4318 \text{ in.}^3} = 21.2 \text{ k} \quad \longleftarrow$$

Problem 5.11-10 A steel beam is built up from a W 16 × 77 wide-flange beam and two 10 in. × 1/2 in. cover plates (see figure on the next page). The allowable load in shear on each bolt is 2.1 kips.

What is the required bolt spacing s in the longitudinal direction if the shear force $V = 30$ kips? (*Note:* Obtain the dimensions and moment of inertia of the W shape from Table E-1.)



Solution 5.11-10 Beam with cover plates



All dimensions in inches.
Wide-flange beam (W 16 × 77):
$d = 16.52$ in.
$I_{beam} = 1110$ in.^{4}
Cover plates:
$b = 10$ in. $t = 0.5$ in.
F = allowable load per bolt
= 2.1 k
V = shear force
= 30 k
s = spacing of bolts in the longitudinal direction
Find s_{max}

MOMENT OF INERTIA ABOUT THE NEUTRAL AXIS

$$I = I_{beam} + 2\left[\frac{1}{12}bt^3 + (bt)\left(\frac{d}{2} + \frac{t}{2}\right)^2\right]$$

$$= 1110 \text{ in.}^4 + 2\left[\frac{1}{12}(10)(0.5)^3 + (10)(0.5)(8.51)^2\right]$$

$$= 1834 \text{ in.}^4$$

FIRST MOMENT OF AREA OF A COVER PLATE

$$Q = bt\left(\frac{d+t}{2}\right) = (10)(0.5)(8.51) = 42.55 \text{ in.}^3$$

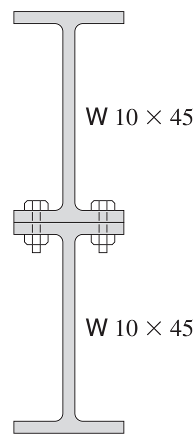
MAXIMUM SPACING OF BOLTS

$$f = \frac{VQ}{I} = \frac{2F}{s} \qquad s = \frac{2FI}{VQ}$$

$$s_{max} = \frac{2(2.1 \text{ k})(1834 \text{ in.}^4)}{(30 \text{ k})(42.55 \text{ in.}^3)} = 6.03 \text{ in.} \quad \leftarrow$$

Problem 5.11-11 Two W 10 × 45 steel wide-flange beams are bolted together to form a built-up beam as shown in the figure.

What is the maximum permissible bolt spacing s if the shear force $V = 20$ kips and the allowable load in shear on each bolt is $F = 3.1$ kips? (*Note:* Obtain the dimensions and properties of the W shapes from Table E-1.)



Solution 5.11-11 Built-up steel beam

All dimensions in inches.
W 10 × 45: $I_1 = 248$ in.^{4} $d = 10.10$ in.
$A = 13.3$ in.^{2}
$V = 20$ k $F = 3.1$ k

Find maximum allowable bolt spacing s_{max}.

MOMENT OF INERTIA OF BUILT-UP BEAM

$$I = 2\left[I_1 + A\left(\frac{d}{2}\right)^2\right] = 2[248 + (13.3)(5.05)^2]$$

$$= 1174.4 \text{ in.}^4$$

FIRST MOMENT OF AREA OF ONE BEAM

$$Q = A\left(\frac{d}{2}\right) = (13.3)(5.05) = 67.165 \text{ in.}^3$$

MAXIMUM SPACING OF BOLTS IN THE LONGITUDINAL DIRECTION

$$f = \frac{VQ}{I} = \frac{2F}{s} \qquad s = \frac{2FI}{VQ}$$

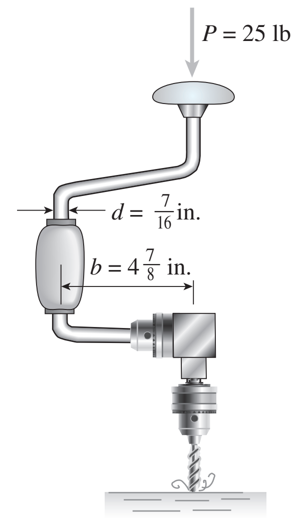
$$s_{max} = \frac{2(3.1 \text{ k})(1174.4 \text{ in.}^4)}{(20 \text{ k})(67.165 \text{ in.}^3)} = 5.42 \text{ in.} \quad \leftarrow$$

Beams with Axial Loads

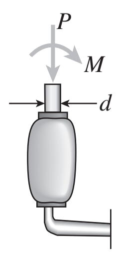
When solving the problems for Section 5.12, assume that the bending moments are not affected by the presence of lateral deflections.

Problem 5.12-1 While drilling a hole with a brace and bit, you exert a downward force $P = 25$ lb on the handle of the brace (see figure). The diameter of the crank arm is $d = 7/16$ in. and its lateral offset is $b = 4\text{-}7/8$ in.

Determine the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the crank.



Solution 5.12-1 **Brace and bit**



$P = 25$ lb (compression)

$M = Pb = (25 \text{ lb})(4\ 7/8 \text{ in.})$

$= 121.9$ lb-in.

$d =$ diameter

$d = 7/16$ in.

$$A = \frac{\pi d^2}{4} = 0.1503 \text{ in.}^2$$

$$S = \frac{\pi d^3}{32} = 0.008221 \text{ in.}^3$$

Maximum stresses

$$\sigma_t = -\frac{P}{A} + \frac{M}{S} = -\frac{25 \text{ lb}}{0.1503 \text{ in.}^2} + \frac{121.9 \text{ lb-in.}}{0.008221 \text{ in.}^3}$$

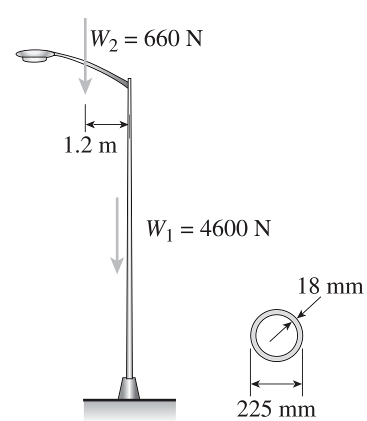
$$= -166 \text{ psi} + 14{,}828 \text{ psi} = 14{,}660 \text{ psi} \longleftarrow$$

$$\sigma_c = -\frac{P}{A} - \frac{M}{S} = -166 \text{ psi} - 14{,}828 \text{ psi}$$

$$= -14{,}990 \text{ psi} \longleftarrow$$

Problem 5.12-2 An aluminum pole for a street light weighs 4600 N and supports an arm that weighs 660 N (see figure). The center of gravity of the arm is 1.2 m from the axis of the pole. The outside diameter of the pole (at its base) is 225 mm and its thickness is 18 mm.

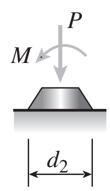
Determine the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the pole (at its base) due to the weights.



Solution 5.12-2 Aluminum pole for a street light

W_1 = weight of pole
= 4600 N
W_2 = weight of arm
= 660 N
b = distance between axis of pole and center of gravity of arm
= 1.2 m
d_2 = outer diameter of pole = 225 mm
d_1 = inner diameter of pole
= 225 mm − 2(18 mm) = 189 mm

AT BASE OF POLE



$P = W_1 + W_2 = 5260$ N
$M = W_2 b = 792$ N · m

PROPERTIES OF THE CROSS SECTION

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 11{,}706 \text{ mm}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 63.17 \times 10^6 \text{ mm}^4$$

$$c = \frac{d_2}{2} = 112.5 \text{ mm}$$

MAXIMUM STRESSES

$$\sigma_t = -\frac{P}{A} + \frac{Mc}{I} = -\frac{5260 \text{ N}}{11{,}706 \text{ mm}^2} + \frac{(792 \text{ N}\cdot\text{m})(112.5 \text{ mm})}{63.17 \times 10^6 \text{ mm}^4}$$

$$= -0.4493 \text{ MPa} + 1.4105 \text{ MPa}$$

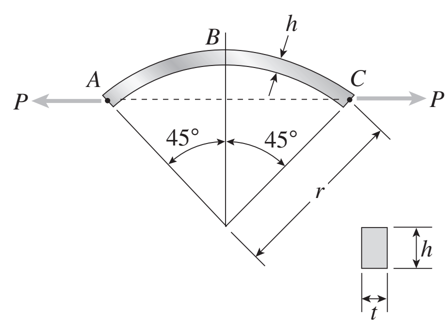
$$= 0.961 \text{ MPa} = 961 \text{ kPa} \quad \leftarrow$$

$$\sigma_c = -\frac{P}{A} - \frac{Mc}{I} = -0.4493 \text{ MPa} - 1.4105 \text{ MPa}$$

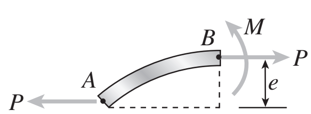
$$= -1.860 \text{ MPa} = -1860 \text{ kPa} \quad \leftarrow$$

Problem 5.12-3 A curved bar *ABC* having a circular axis (radius $r = 12$ in.) is loaded by forces $P = 400$ lb (see figure). The cross section of the bar is rectangular with height h and thickness t.

If the allowable tensile stress in the bar is 12,000 psi and the height $h = 1.25$ in., what is the minimum required thickness t_{min}?



Solution 5.12-3 Curved bar



r = radius of curved bar

$$e = r - r\cos 45°$$

$$= r\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$M = Pe = \frac{Pr}{2}(2 - \sqrt{2})$$

CROSS SECTION

h = height $\quad t$ = thickness $\quad A = ht \quad S = \frac{1}{6}th^2$

TENSILE STRESS

$$\sigma_t = \frac{P}{A} + \frac{M}{S} = \frac{P}{ht} + \frac{3Pr(2 - \sqrt{2})}{th^2}$$

$$= \frac{P}{ht}\left[1 + 3(2 - \sqrt{2})\frac{r}{h}\right]$$

MINIMUM THICKNESS

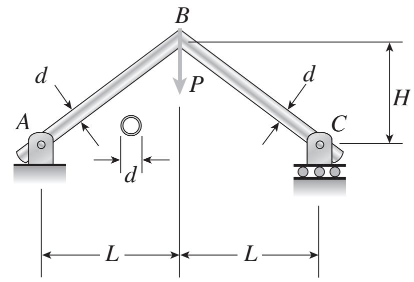
$$t_{min} = \frac{P}{h\sigma_{allow}}\left[1 + 3(2 - \sqrt{2})\frac{r}{h}\right]$$

SUBSTITUTE NUMERICAL VALUES:

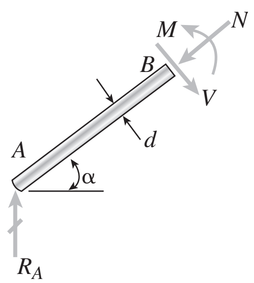
$P = 400$ lb $\quad \sigma_{allow} = 12{,}000$ psi
$r = 12$ in. $\quad h = 1.25$ in.
$t_{min} = 0.477$ in. $\leftarrow$

Problem 5.12-4 A rigid frame *ABC* is formed by welding two steel pipes at *B* (see figure). Each pipe has cross-sectional area $A = 11.31 \times 10^3$ mm^2, moment of inertia $I = 46.37 \times 10^6$ mm^4, and outside diameter $d = 200$ mm.

Find the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the frame due to the load $P = 8.0$ kN if $L = H = 1.4$ m.



Solution 5.12-4 **Rigid frame**



Load *P* at midpoint *B*

REACTIONS: $R_A = R_C = \dfrac{P}{2}$

BAR *AB*:

$$\tan \alpha = \frac{H}{L}$$

$$\sin \alpha = \frac{H}{\sqrt{H^2 + L^2}}$$

d = diameter
$c = d/2$

AXIAL FORCE: $N = R_A \sin \alpha = \dfrac{P}{2} \sin \alpha$

BENDING MOMENT: $M = R_A L = \dfrac{PL}{2}$

TENSILE STRESS

$$\sigma_t = -\frac{N}{A} + \frac{Mc}{I} = -\frac{P \sin \alpha}{2A} + \frac{PLd}{4I}$$

SUBSTITUTE NUMERICAL VALUES:

$P = 8.0$ kN $\quad L = H = 1.4$ m $\quad \alpha = 45°$
$\sin \alpha = 1/\sqrt{2} \quad d = 200$ mm
$A = 11.31 \times 10^3$ mm$^2 \quad I = 46.37 \times 10^6$ mm^4

$$\sigma_t = -\frac{(8.0 \text{ kN})(1/\sqrt{2})}{2(11.31 \times 10^3 \text{ mm}^2)} + \frac{(8.0 \text{ kN})(1.4 \text{ m})(200 \text{ mm})}{4(46.37 \times 10^6 \text{ mm}^4)}$$

$$= -0.250 \text{ MPa} + 12.08 \text{ MPa}$$

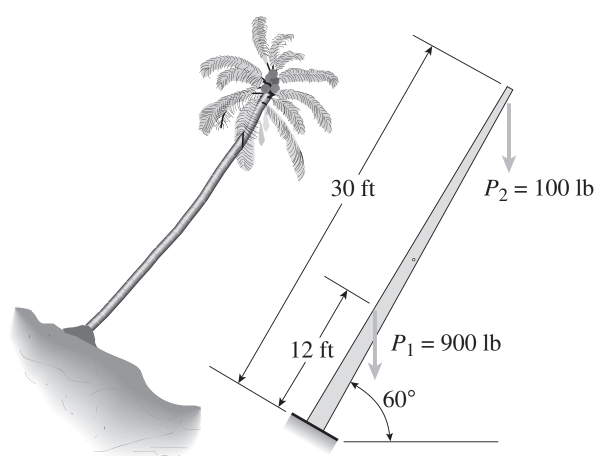
$$= 11.83 \text{ MPa (tension)} \quad \longleftarrow$$

$$\sigma_c = -\frac{N}{A} - \frac{Mc}{I} = -0.250 \text{ MPa} - 12.08 \text{ MPa}$$

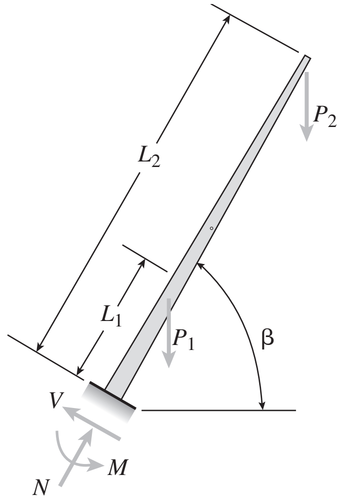
$$= -12.33 \text{ MPa (compression)} \quad \longleftarrow$$

Problem 5.12-5 A palm tree weighing 1000 lb is inclined at an angle of 60° (see figure). The weight of the tree may be resolved into two resultant forces, a force $P_1 = 900$ lb acting at a point 12 ft from the base and a force $P_2 = 100$ lb acting at the top of the tree, which is 30 ft long. The diameter at the base of the tree is 14 in.

Calculate the maximum tensile and compressive stresses σ_t and σ_c, respectively, at the base of the tree due to its weight.



Solution 5.12-5 Palm tree



FREE-BODY DIAGRAM

$P_1 = 900$ lb
$P_2 = 100$ lb
$L_1 = 12$ ft $= 144$ in.
$L_2 = 30$ ft $= 360$ in.
$d = 14$ in.

$$A = \frac{\pi d^2}{4} = 153.94 \text{ in.}^2$$

$$S = \frac{\pi d^3}{32} = 269.39 \text{ in.}^3$$

$$M = P_1L_1 \cos 60° + P_2L_2 \cos 60°$$
$$= [(900 \text{ lb})(144 \text{ in.}) + (100 \text{ lb})(360 \text{ in.})] \cos 60°$$
$$= 82{,}800 \text{ lb-in.}$$
$$N = (P_1 + P_2) \sin 60° = (1000 \text{ lb}) \sin 60° = 866 \text{ lb}$$

MAXIMUM TENSILE STRESS

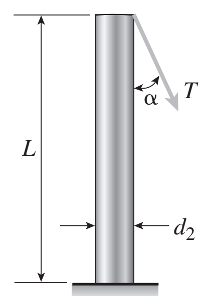
$$\sigma_t = -\frac{N}{A} + \frac{M}{S} = -\frac{866 \text{ lb}}{153.94 \text{ in.}^2} + \frac{82{,}800 \text{ lb-in.}}{269.39 \text{ in.}^3}$$
$$= -5.6 \text{ psi} + 307.4 \text{ psi} = 302 \text{ psi} \quad \leftarrow$$

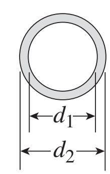
MAXIMUM COMPRESSIVE STRESS

$$\sigma_c = -5.6 \text{ psi} - 307.4 \text{ psi} = -313 \text{ psi} \quad \leftarrow$$

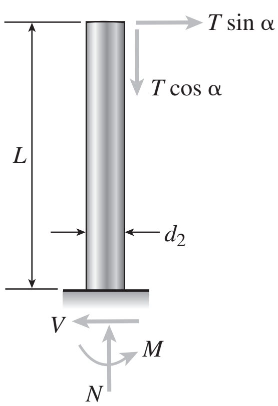
Problem 5.12-6 A vertical pole of aluminum is fixed at the base and pulled at the top by a cable having a tensile force T (see figure). The cable is attached at the outer surface of the pole and makes an angle $\alpha = 25°$ at the point of attachment. The pole has length $L = 2.0$ m and a hollow circular cross section with outer diameter $d_2 = 260$ mm and inner diameter $d_1 = 200$ mm.

Determine the allowable tensile force T_{allow} in the cable if the allowable compressive stress in the aluminum pole is 90 MPa.





Solution 5.12-6 Aluminum pole



$\alpha = 25°$
$L = 2.0$ m
$d_2 = 260$ mm
$d_1 = 200$ mm
$(\sigma_c)_{allow} = 90$ MPa

CROSS SECTION

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 21{,}677 \text{ mm}^2 = 21.677 \times 10^{-3} \text{ mm}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 145{,}778 \times 10^3 \text{ mm}^4$$
$$= 145.778 \times 10^{-6} \text{ m}^4$$

$$c = \frac{d_2}{2} = 130 \text{ mm} = 0.13 \text{ m}$$

AT THE BASE OF THE POLE

$$N = T \cos \alpha = 0.90631T \qquad (N, T = \text{newtons})$$

$$M = (T \cos \alpha)\left(\frac{d_2}{2}\right) + (T \sin \alpha)(L)$$
$$= 0.11782\,T + 0.84524\,T$$
$$= 0.96306\,T \qquad (M = \text{newton meters})$$

COMPRESSIVE STRESS

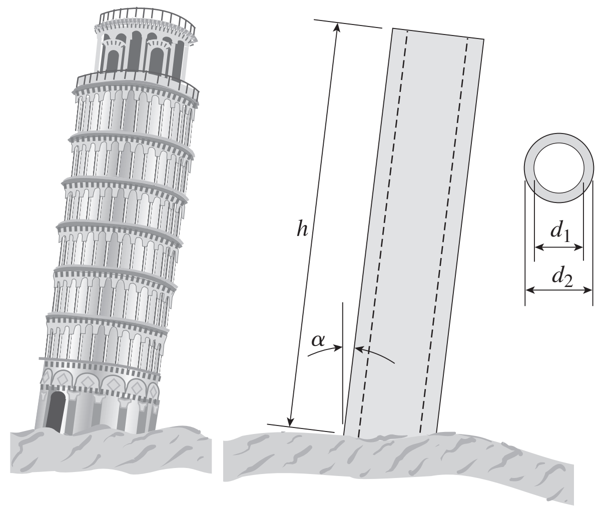
$$\sigma_c = \frac{N}{A} + \frac{Mc}{I} = \frac{0.90631T}{21.677 \times 10^{-3}\ \text{m}^2} + \frac{(0.96306T)(0.13\ \text{m})}{145.778 \times 10^{-6}\ \text{m}^4}$$
$$= 41.82\,T + 858.83\,T$$
$$= 900.64\,T \qquad (\sigma_c = \text{pascals})$$

ALLOWABLE TENSILE FORCE

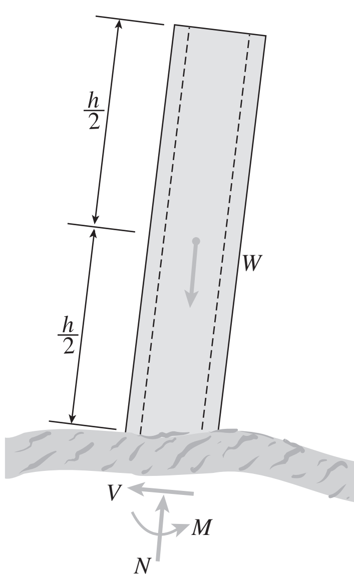
$$T_{\text{allow}} = \frac{(\sigma_c)_{\text{allow}}}{900.64} = \frac{90 \times 10^6\ \text{pascals}}{900.64}$$
$$= 99{,}900\ \text{N} = 99.9\ \text{kN} \longleftarrow$$

Problem 5.12-7 Because of foundation settlement, a circular tower is leaning at an angle α to the vertical (see figure). The structural core of the tower is a circular cylinder of height h, outer diameter d_2, and inner diameter d_1. For simplicity in the analysis, assume that the weight of the tower is uniformly distributed along the height.

Obtain a formula for the maximum permissible angle α if there is to be no tensile stress in the tower.



Solution 5.12-7 Leaning tower



W = weight of tower
α = angle of tilt

CROSS SECTION

$$A = \frac{\pi}{4}(d_2^2 - d_1^2)$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4)$$
$$= \frac{\pi}{64}(d_2^2 - d_1^2)(d_2^2 + d_1^2)$$

$$\frac{I}{A} = \frac{d_2^2 + d_1^2}{16}$$

$$c = \frac{d_2}{2}$$

AT THE BASE OF THE TOWER

$$N = W\cos\alpha \qquad M = W\left(\frac{h}{2}\right)\sin\alpha$$

TENSILE STRESS (EQUAL TO ZERO)

$$\sigma_t = -\frac{N}{A} + \frac{Mc}{I} = -\frac{W\cos\alpha}{A} + \frac{W}{I}\left(\frac{h}{2}\sin\alpha\right)\left(\frac{d_2}{2}\right) = 0$$

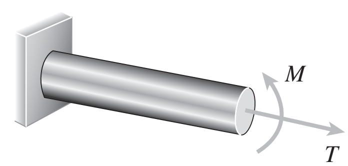
$$\therefore\ \frac{\cos\alpha}{A} = \frac{hd_2\sin\alpha}{4I} \qquad \tan\alpha = \frac{4I}{hd_2A} = \frac{d_2^2 + d_1^2}{4hd_2}$$

MAXIMUM ANGLE α

$$\alpha = \arctan\frac{d_2^2 + d_1^2}{4hd_2} \longleftarrow$$

Problem 5.12-8 A steel bar of solid circular cross section is subjected to an axial tensile force $T = 26$ kN and a bending moment $M = 3.2$ kN · m (see figure).

Based upon an allowable stress in tension of 120 MPa, determine the required diameter d of the bar. (Disregard the weight of the bar itself.)



Solution 5.12-8 Circular bar

$T = 26$ kN $M = 3.2$ kN · m

$\sigma_{allow} = 120$ MPa d = diameter

$$A = \frac{\pi d^2}{4} \qquad S = \frac{\pi d^3}{32}$$

TENSILE STRESS

$$\sigma_t = \frac{T}{A} + \frac{M}{S} = \frac{4T}{\pi d^2} + \frac{32M}{\pi d^3}$$

or $\pi d^3 \sigma_{allow} - 4Td - 32M = 0$

$(\pi)(120 \text{ MPa})d^3 - 4(26 \text{ kN})d - 32(3.2 \text{ kN} \cdot \text{m}) = 0$

(d = meters)

$(120{,}000{,}000 \text{ N/m}^2)(\pi)d^3 - (104{,}000 \text{ N})d$
$- 102{,}400 \text{ N} \cdot \text{m} = 0$

SIMPLIFY THE EQUATION:

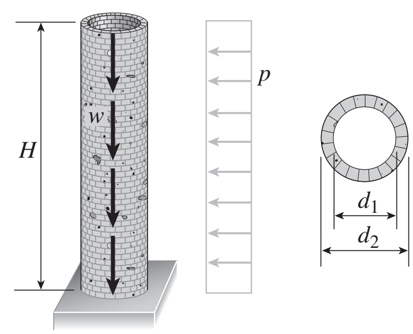
$(15{,}000\ \pi)\ d^3 - 13d - 12.8 = 0$

SOLVE NUMERICALLY FOR THE REQUIRED DIAMETER:

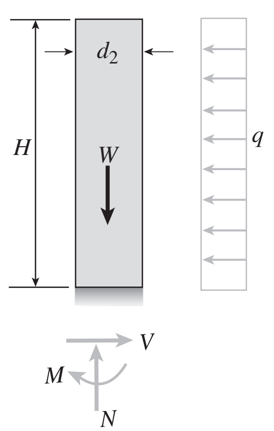
$d = 0.0662 \text{ m} = 66.2 \text{ mm}$ ←

Problem 5.12-9 A cylindrical brick chimney of height H weighs $w = 825$ lb/ft of height (see figure). The inner and outer diameters are $d_1 = 3$ ft and $d_2 = 4$ ft, respectively. The wind pressure against the side of the chimney is $p = 10$ lb/ft^2 of projected area.

Determine the maximum height H if there is to be no tension in the brickwork.



Solution 5.12-9 Brick chimney



p = wind pressure
q = intensity of load = pd_2
d_2 = outer diameter
d_1 = inner diameter
W = total weight of chimney = wH

CROSS SECTION

$$A = \frac{\pi}{4}(d_2^2 - d_1^2)$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = \frac{\pi}{64}(d_2^2 - d_1^2)(d_2^2 + d_1^2)$$

$$\frac{I}{A} = \frac{1}{16}(d_2^2 + d_1^2) \qquad c = \frac{d_2}{2}$$

AT BASE OF CHIMNEY

$$N = W = wH \qquad M = qH\left(\frac{H}{2}\right) = \frac{1}{2}pd_2H^2$$

TENSILE STRESS (EQUAL TO ZERO)

$$\sigma_t = -\frac{N}{A} + \frac{Md_2}{2I} = 0 \qquad \text{or} \qquad \frac{M}{N} = \frac{2I}{Ad_2}$$

$$\frac{pd_2H^2}{2wH} = \frac{d_2^2 + d_1^2}{8d_2}$$

SOLVE FOR H $H = \dfrac{w(d_2^2 + d_1^2)}{4pd_2^2}$ ←

SUBSTITUTE NUMERICAL VALUES

$w = 825$ lb/ft $d_2 = 4$ ft $d_1 = 3$ ft $p = 10$ lb/ft^2

$H_{max} = 32.2$ ft ←